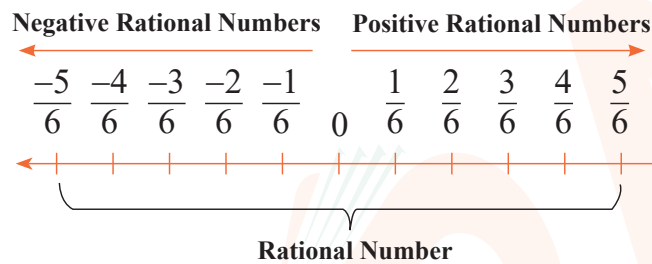


Rational Number



A number which can be written in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$ is called a rational number.

Example: $-\frac{2}{3}, \frac{6}{7}$



Operations on Rational Numbers

(a) Addition of Rational Numbers

Case 1: Same Denominator

Add the numerators, Keep the denominator same and Simplify the fraction if necessary.

Example: $\frac{3}{4} + \frac{2}{4} = \frac{3+2}{4} = \frac{5}{4}$

Case 2: Different Denominators

Find the Least Common Multiple (LCM), Convert the fractions to have the same denominator, Add the numerators while keeping the denominator same and Simplify the fraction if necessary.

Example: $\frac{1}{5} + \frac{2}{7} = \frac{1 \times 7}{5 \times 7} + \frac{2 \times 5}{7 \times 5} = \frac{7}{35} + \frac{10}{35} = \frac{17}{35}$

(b) Subtraction of Rational Numbers

The method for subtraction is similar to addition, with the only difference being that we subtract the numerators instead of adding them. We can follow the same examples as we did for addition.

(c) Multiplication of Rational Numbers

Multiply the numerators together, Multiply the denominators together and Simplify the fraction if possible.

Example: $\frac{4}{3} \times \frac{2}{5} = \frac{4 \times 2}{3 \times 5} = \frac{8}{15}$

(d) Division of Rational Numbers

Invert (reciprocal) the divisor, Multiply the first fraction by the reciprocal of the second fraction and Simplify if necessary.

Example: $\frac{7}{9} \div \frac{2}{3} = \frac{7}{9} \times \frac{3}{2} = \frac{7 \times 3}{9 \times 2} = \frac{21}{18} = \frac{7}{6}$

Properties of Rational Numbers

(a) Closure Property: When we perform any operation on a rational number, such that the resultant also belong to the same set then we say it follows closure property.

Example: For addition: $\frac{1}{15} + \left(-\frac{1}{15}\right) = 0$, which is also a rational number. So, it is closed under addition.

So for operations like subtraction, multiplication and division (except when divisor is 0) of rational number are also closed under closure property.

(b) Commutative Property: An arithmetic operation is considered commutative if swapping the order of its operands does not alter the outcome. If any operation on two rational numbers satisfies the above-mentioned property, we say it is commutative under that particular operation.

Example: For addition $\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$ and $\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$, Here, $\frac{1}{5} + \frac{2}{5} = \frac{2}{5} + \frac{1}{5}$ So, it is commutative under addition.

Commutative property holds for addition and multiplication and it does not hold for Subtraction and division.

(c) Associative Property: A set of numbers is said to follow associative property over a particular operation if even after changing the grouping of three numbers, we get the same result. If any operation of a rational number satisfies the above mentioned property, we say it is associative under that particular operation.

Example: For addition $\left(-\frac{2}{5} + \frac{2}{5}\right) + 1 = 0 + 1 = 1$ and $-\frac{2}{5} + \left(\frac{2}{5} + 1\right) = -\frac{2}{5} + \frac{7}{5} = 1$

Here, $\left(-\frac{2}{5} + \frac{2}{5}\right) + 1 = -\frac{2}{5} + \left(\frac{2}{5} + 1\right)$. So, it is associative under addition.

Associative property holds for addition and multiplication and it does not hold for Subtraction and Division.

(d) Distributive Property: For any rational numbers a, b and c, then $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$. This above property is called Distributive Property.

Rational numbers hold the distributive property for multiplication over addition and subtraction.

Additive Identity of Rational Numbers

When 0 is added to a number, the sum is the same number. Therefore, 0 is called the identity for the addition of rational numbers. It is the additive identity for integers and whole numbers also.

Example: $\left(-\frac{5}{7}\right) + 0 = 0 + \left(-\frac{5}{7}\right) = -\frac{5}{7}$

Additive Inverse of Rational Numbers

The additive inverse property states that for every number 'a', there is a corresponding number '-a', known as its additive inverse, such that their sum is always zero (additive identity). In other words, $a + (-a) = 0$ (Additive Identity)

Example: $\left(\frac{2}{3}\right) + \left(-\frac{2}{3}\right) = 0$

Multiplicative Identity of Rational Numbers

The multiplicative identity is a number that, when multiplied by any other number in a given set, leaves the original number unchanged.

For rational numbers, the multiplicative identity is 1 because multiplying any rational number by 1 result in the same number.

Example: $\frac{3}{4} \times 1 = \frac{3}{4}$

Multiplicative Inverse of Rational Numbers

For any non-zero rational number $\frac{a}{b}$ there exist a rational number $\frac{c}{d}$, such that $\frac{a}{b} \times \frac{c}{d} = 1$, where $\frac{c}{d}$ is called the reciprocal or multiplicative inverse of $\frac{a}{b}$

Practice Questions

1. Evaluate $\frac{12}{7} - \frac{5}{7}$

Sol: $\frac{12}{7} - \frac{5}{7} = \frac{12-5}{7} = \frac{7}{7} = 1$

2. The multiplicative inverse of $-1\frac{2}{3}$ is:

Sol: The multiplicative inverse of $-1\frac{2}{3} = \frac{-3}{5}$