

Square and Square Root



Square : The square of a number is that number raised to the power 2.

Example: $13^2 = 13 \times 13 = 169$ so, we can say that the square of 13 is 169.

Perfect Square: Number that can be expressed as the square of an integer.

Example: 16 is a perfect square because $4^2 = 16$.

Properties of Square Numbers

1. A perfect square never ends in 2, 3, 7, or 8 at the unit's place.
2. A natural number ending in 0, 1, 4, 5, 6, or 9 may or may not be a perfect square.
3. If a number has 0 in the unit place, then its square ends in 0
4. A perfect square always ends in an even number of zeros and if it ends in an odd number of zeros, it is never a perfect square.
5. If a number has 1 or 9 in the unit place, then its square ends in 1.
6. If a number has 2 or 8 in the unit place, then its square ends in 4.
7. If a number has 3 or 7 in the unit place, then its square ends in 9.
8. If a number has 4 or 6 in the unit place, then its square ends in 6.
9. If a number has 5 in the unit place, then its square ends in 5.
10. Square of an odd number is odd and of even number is even.

Important Patterns

- (a) There are $2n$ non-perfect square numbers between two consecutive square numbers n^2 and $(n + 1)^2$
- (b) The sum of first n odd natural numbers is n^2
- (c) The square of any odd natural number other than 1 can be expressed as the sum of two consecutive numbers which can be given by

$$\left[\left(\frac{n^2 - 1}{2} \right) + \left(\frac{n^2 + 1}{2} \right) \right]$$

Finding a Square of a Number: To find the square of a number, simply multiply the number by itself.

Example: To find the square of 17, you multiply 17 by 17: $17 \times 17 = 289$. So, the square of 17 is 289.

Pythagorean triplet: Consists of three positive integers a, b, and c such that $a^2 + b^2 = c^2$.

Example, 3, 4, 5 is a Pythagorean triplet because $3^2 + 4^2 = 9 + 16 = 25 = 5^2$.

Square root: The square root of a number is a value that, when multiplied by itself gives the original number. For example, the square root of 196 is 14 because $14 \times 14 = 196$

(a) Finding square root through prime factorization: Express the number as a product of prime factors and then pair up the factors. For example: for 36, the prime factorization is $36 = 2^2 \times 3^2$, so the square root is $2 \times 3 = 6$

(b) Finding Square Root through Long Division Method: It involves dividing the number into pairs of digits, then progressively subtracting and bringing down pairs to estimate the root.

Example:

	225
2	506 25
	4
42	106
	84
445	2225
	2225
	0

(c) Estimating Square Root: It involves finding two perfect squares between which the number lies.

Example: The square root of 50 is between $7(7^2 = 49)$ and $8(8^2 = 64)$.

(d) Square Roots of decimals: It can be found by applying the same methods such as prime factorization or long division on decimal numbers.

Example: the square root of 0.49 is 0.7.

(e) Square Root of Product of Two Numbers: The square root of the product of two numbers is equal to the product of their square roots.

Example: $\sqrt{16 \times 25} = \sqrt{16} \times \sqrt{25} = 4 \times 5 = 20$

Practice Questions

1. A number added to its square gives 72. What is the number?

Sol: Number is 4.

$$8 + 8^2 = 8 + 64 = 72$$

2. How many natural numbers lie between 15^2 and 16^2 ?

Sol: Natural numbers lie between 15^2 and $16^2 = 2 \times 15 = 30$.

3. Find the length of the side of a square whose area is 625 cm^2

Sol: Area of square = (Side)²

$$625 = (\text{Side})^2$$

$$\sqrt{625} = \text{Side}$$

$$\text{Side} = 25 \text{ cm}$$

4. The value of $\sqrt{318 + \sqrt{25 + \sqrt{121}}}$ is:

Sol: $\sqrt{318 + \sqrt{25 + \sqrt{121}}}$

$$\sqrt{318 + \sqrt{25 + 11}}$$

$$\sqrt{318 + \sqrt{36}}$$

$$\sqrt{318 + 6}$$

$$\sqrt{324} = 18$$

