

Cube and Cube Root



Cube

The cube of a number is that number raised to the power 3 .

Example: $3^3 = 3 \times 3 \times 3 = 27$. Thus, cube of 3 is 27.

Perfect Cube: A number that can be expressed as the cube of an integer. In other words, a number n is a perfect cube if there exists an integer m such that $n = m^3$.

Example: 125 is a perfect cube because $5^3 = 125$.

Properties of Cube of Numbers

- (i) Cube of all even numbers is even.
- (ii) Cube of all odd numbers is odd.
- (iii) The sum of the cubes of first n natural numbers is equal to the square of their sum.

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = (1 + 2 + 3 + \cdots + n)^2 = \left(\frac{n(n+1)}{2} \right)^2$$

- (iv) Cube of the digits 1 to 9.

x	1	2	3	4	5	6	7	8	9
x^3	1	8	27	64	125	216	343	512	729

If the number ends in 1,4,5,6,9 then its cube root also ends in 1,4,5,6,9.

Cube of 2 ends in 8 and cube of 8 ends in 2 .

Cube of 3 ends in 7 and cube of 7 ends in 3.

- (v) The cube of negative integers is always negative.

Cube root

The cube root of a number is the value that, when multiplied by itself three times gives the original number. It is represented as $\sqrt[3]{n}$, where n is the number whose cube root is being calculated.

Example: The cube root of 343 is 7 because $7 \times 7 \times 7 = 343$. So, $\sqrt[3]{343} = 7$

(a) Cube Root through Prime Factorization Method: To find the cube root using prime factorization, express the number as a product of prime factors and group them into triples.

Example: for 216, the prime factorization is $2^3 \times 3^3$, so the cube root is $2 \times 3 = 6$.

(b) Finding Cube Root through Estimation:

Let us take example of 15625

1. Take any cube number and start making groups of three digits starting from the right
2. The first group will give the unit place digit.
3. Take the second group. Find the number whose cube is less than or equal to this number. It will give the tens digit of the cube

Obtain the required cube root by forming a number whose unit digit is the number obtained from first group and tens digit is the number obtained from second group

15 - second group

625 - first group

625 ends with 5. The unit place digit of this cube root is 5.

$$2^3 = 8 \text{ and } 3^3 = 27$$

$$8 < 15 < 27$$

$$2^3 < 15 < 3^3$$

The tens digit of the cube root is 2

$$\sqrt[3]{15625} = 25$$

(c) Cube Root of the Product of Integers: The cube root of the product of two integers is equal to the product of their cube roots.

$$\text{Example: } \sqrt[3]{64 \times 27} = \sqrt[3]{64} \times \sqrt[3]{27} = 4 \times 3 = 12$$

Practice Questions

1. The volume of a cube is 729 cm^3 . The edge of the cube is:

Sol: The volume of a cube = $(\text{Side})^3$

$$729 = (\text{Side})^3$$

$$\sqrt[3]{729} = \text{Side}$$

$$\text{Side} = 9 \text{ cm}$$

2. The cube of 0.8 is:

$$\text{Sol: Cube of } 0.8 = (0.8)^3 = 0.8 \times 0.8 \times 0.8 = 0.512$$

3. By what least number should 40824 be divided to get a perfect cube?

Sol: Prime factorization of 40824

2	40824
2	20412
2	10206
3	5103
3	1701
3	567
3	189
3	63
3	21
7	7
	1

Making the triplets, we find that three triplet are formed and we are left with one more factor. Hence, 40824 to be divided by 7 to get a perfect cube.