

Exponents and Power



Exponent

An exponent represents how many times a number (called the base) is multiplied by itself.

If a is any number and n is a positive integer, then:

$$a^n = a \times a \times a \times \dots \times a \text{ (} n \text{ times)}$$

Here a is called the base, n is called the exponent and a^n is called a raised to the power n or just a to the power n .

Example: $2^4 = 2 \times 2 \times 2 \times 2 = 16$, $5^3 = 5 \times 5 \times 5 = 125$

Laws of Exponents

1. Product Law: $a^m \times a^n = a^{m+n}$

Example: $2^3 \times 2^4 = 2^{3+4} = 2^7 = 128$

2. Quotient Law (when base is same): $\frac{a^m}{a^n} = a^{m-n} \text{ (} a \neq 0 \text{)}$

Example: $\frac{5^6}{5^2} = 5^{6-2} = 5^4 = 625$

3. Power of a Power Law: $(a^m)^n = a^{m \times n}$

Example: $(3^2)^4 = 3^{2 \times 4} = 3^8 = 6561$

4. Power of a Product Law: $(ab)^m = a^m \times b^m$

Example: $(2 \times 5)^3 = 2^3 \times 5^3 = 8 \times 125 = 1000$

5. Power of a Quotient Law: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \text{ (} b \neq 0 \text{)}$

Example: $\left(\frac{4}{2}\right)^2 = \frac{4^2}{2^2} = \frac{16}{4} = 4$

6. Zero Exponent Law: $a^0 = 1 \text{ (} a \neq 0 \text{)}$

Example: $9^0 = 1$

7. Negative Exponent Law: $a^{-m} = \frac{1}{a^m} (a \neq 0)$

Example: $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

Expressing Numbers in Standard Form

Standard form (also called scientific notation) is a way of writing very large or very small numbers in the form: $a \times 10^n$

Where: $1 \leq a < 10$ (i.e., a is a number between 1 and 10), n is an integer (positive or negative) and the number is written as a product of a number and a power of 10.

Examples:

1. Large number: $3000000 = 3 \times 10^6$ (Move the decimal point 6 places to the left)

2. Small number: $0.00042 = 4.2 \times 10^{-4}$ (Move the decimal point 4 places to the right)

Conversion of Number to Standard Form

For large numbers: Place the decimal after the first non-zero digit, Count how many places you moved the decimal towards the left side that's the power of 10 and the power is positive.

Example: $72,000 = 7.2 \times 10^4$

For small numbers: Place the decimal after the first non-zero digit, Count how many places you moved the decimal towards the right side that's the power of 10 and the power is negative.

Example: $0.0053 = 5.3 \times 10^{-3}$

Practice Questions

1. The expression $(4^{-1} + 5^{-1} + 8^{-1})^0$ is equal to:

Sol: $(4^{-1} + 5^{-1} + 8^{-1})^0 = 1$

2. 3.5×10^{-6} is equal to:

Sol: $3.5 \times 10^{-6} = 0.0000035$

3. The standard form for 794000000 is:

Sol: Standard form of 794000000 is 7.94×10^8 .

4. The value of x for which $\left(\frac{5}{8}\right)^{-4} \times \left(\frac{5}{8}\right)^{3x} = \left(\frac{5}{8}\right)^5$ is:

Sol: $\left(\frac{5}{8}\right)^{-4} \times \left(\frac{5}{8}\right)^{3x} = \left(\frac{5}{8}\right)^5$

$$\left(\frac{5}{8}\right)^{-4+3x} = \left(\frac{5}{8}\right)^5$$

$$-4 + 3x = 5$$

$$3x = 9$$

$$x = 3$$