

Probability



Experiment: An action or process that leads to one or more outcomes.

Example: Tossing a coin.

Outcome: A possible result of an experiment.

Example: Getting Heads in a coin toss.

Sample Space: The set of all possible outcomes of an experiment.

Example: For a die roll, sample space = {1, 2, 3, 4, 5, 6}.

Event: A specific outcome or a group of outcomes from the sample space.

Example: Getting an even number on a die (Event = {2, 4, 6}).

Favorable Outcome: The outcomes that satisfy the condition of a desired event.

Example: For event getting a 4 when rolling a die, the favorable outcome = {4}.

Probability: Probability is the measure of how likely an event is to occur, ranging from 0 (impossible) to 1 (certain).

$$\text{Probability of an event} = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$

Example: Probability of getting a 5 on a die = $\frac{1}{6}$

Sample spaces for tossing a coin

1. Throwing a coin once:

Sample Space (S) = {H, T}

2. Throwing a coin twice:

Sample Space (S) = {HH, HT, TH, TT}

Sample spaces of rolling a dice

1. Rolling a die once

Sample Space (S) = {1, 2, 3, 4, 5, 6}

2. Rolling a die twice:

Sample Space (S) = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

Properties of probability

1. Probability lies between 0 and 1.
2. Sum of probabilities of all outcomes in a sample space is 1.
3. Probability of an impossible event is 0.
4. Probability of a certain event is 1.
5. For any event A, $P(\text{not } A) = 1 - P(A)$

Practice questions

1. A die is rolled. What is the probability of getting a number less than 4?

Sol: Probability of getting a number less than 4

$$\text{Total outcomes} = \{1, 2, 3, 4, 5, 6\} = 6$$

$$\text{Favourable outcomes} = \{1, 2, 3\} = 3$$

$$P = \frac{3}{6} = \frac{1}{2}$$

2. A card is drawn randomly from a deck of 52 cards. What is the probability of drawing a red card?

Sol: Probability of drawing a red card

$$\text{Total cards} = 52$$

$$\text{Red cards (Hearts + Diamonds)} = 26$$

$$P = \frac{26}{52} = \frac{1}{2}$$

3. A die is rolled twice. What is the probability of getting a doublet?

Sol: Probability of getting a doublet (die rolled twice)

$$\text{Total outcomes} = 6 \times 6 = 36$$

$$\text{Favourable outcomes} = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\} = 6$$

$$P = \frac{6}{36} = \frac{1}{6}$$

4. A coin is tossed twice. What is the probability of getting exactly two head?

Sol: Probability of getting exactly two heads

$$\text{Total outcomes} = \{HH, HT, TH, TT\} = 4$$

$$\text{Favourable outcomes} = \{HH\} = 1$$

$$P = \frac{1}{4}$$

5. A bag contains 3 red balls, 5 green balls, and 2 blue balls. What is the probability of drawing a red ball?

Sol: Probability of drawing a red ball

$$\text{Total balls} = 3 + 5 + 2 = 10$$

$$\text{Red balls} = 3$$

$$P = \frac{3}{10} = \frac{3}{10}$$

