

FACTORS AND MULTIPLES



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Factors: Any number which is a perfect divisor of a given number is called a factor of the given number.

For example, the factors of 6 are 1, 2, 3 and 6.

Important Results:

- (a) 1 is a factor of every number.
- (b) Every number is a multiple of itself.
- (c) Every multiple of a number is always less than or equal to the number itself.
- (d) Every number has a finite number of multiples.

Multiples: Just as 1, 2, 3, and 6 are factors of 6, we say that 6 is a multiple of 1, 2, 3, and 6. Every number is a multiple of each of its factors.

Important Results:

- (a) Every number is a multiple of itself.
- (b) Every multiple of a number is equal to or greater than that number.
- (c) Every number has an infinite number of multiples.

$$\begin{array}{ccc} & & \nearrow \text{Multiple of 4} \\ 5 \times 4 = 20 & & \\ \nwarrow \text{Factor of 20} & & \searrow \text{Multiple of 5} \end{array}$$

TYPES OF NUMBERS

- (a) **Even Numbers:** A number which is divisible by 2 is called an even number. Examples of even numbers are: 0, 12, 34, 56, 78....
- (b) **Odd Number** A number which is not exactly divisible by 2 is called an odd number. Examples of odd numbers are: 1, 13, 15, 25, 29....
- (c) **Prime Numbers:** A natural number greater than 1, which has no factors except 1 and itself is called a prime number. Examples of prime numbers are: 2, 3, 5, 11, 13, 17, ...
- (d) **Composite Numbers:** A number is composite if it has at least one factor other than 1 and itself. Examples of composite numbers are: 4, 6, 8, 9, 10, 12, 14, ...
- (e) **Twin primes:** Pairs of prime numbers that have a difference of 2 are called twin primes. Examples of twin primes are: (3, 5), (5, 7), (11, 13), (17, 19)

- (f) **Perfect Numbers:** A perfect number is a positive integer that equals the sum of its proper divisors (divisors excluding the number itself). For example, 6 is a perfect number because its proper divisors (1, 2, and 3) sum to 6.
- (g) **Coprime Numbers:** Two numbers are said to be coprime if they do not have a common factor other than 1.

Examples of coprime numbers are: (8, 15); (5, 9); (2, 11)

Note:

- Every even number greater than 4 can be expressed as the sum of two odd prime numbers.
- E.g., $6 = 3 + 3$; $18 = 5 + 13$; $44 = 13 + 31$.
- 1 is neither prime nor composite.
- Every natural number except 1 is either a prime number or a composite number.
- 2 is the only prime number which is even. All other prime numbers are odd.
- Two prime numbers are always coprime.
- Two coprime numbers need not both be prime numbers.

DIVISIBILITY TEST

- (a) **Test for divisibility by 2:** A number is divisible by 2 if its unit digit is even.
For example, 206 is divisible by 2 because 6 is even.
- (b) **Test for divisibility by 3:** A number is divisible by 3 if the sum of all its digits is exactly divisible by 3.
For example, 567 is divisible by 3 because $5 + 6 + 7 = 18$ is divisible by 3.
- (c) **Test for divisibility by 4:** A number is divisible by 4 if the number formed by its last two digits (one and ten) is divisible by 4.
For example, 312 is divisible by 4 because 12 is divisible by 4.
- (d) **Test for divisibility by 5:** A number is divisible by 5 if its unit digit is 0 or 5.
For example, 3075 is divisible by 5 because its unit digit is 5.
- (e) **Test for divisibility by 6:** A number is divisible by 6 if it is divisible by both 2 and 3.
For example, 456 is divisible by 6 because its unit digit is even so it is divisible by 2, and the sum of the digits of 456 ($4+5+6=15$) is divisible by 3. So, 456 is divisible by both 2 and 3, so we can say that it is divisible by 6.
- (f) **Test for divisibility by 7:** A number is exactly divisible by 7 if the number obtained by doubling the last digit of the number and subtracting it from the number formed by the remaining digits is exactly divisible by 7. And this process can be repeated.
- (g) **Test for divisibility by 8:** A number is divisible by 8 if the sum of its last three digits (units, tens and hundreds) is divisible by 8.
For example, 8864 is divisible by 8 because 864 is divisible by 8.

(h) **Test for divisibility by 9:** A number is divisible by 9 if the sum of its digits is divisible by 9.
For example, 4563 is divisible by 9 because the sum of the digits of 4563 ($4 + 5 + 6 + 3 = 18$) is divisible by 9.

(i) **Test for divisibility by 10:** A number is divisible by 10 if its units digit is 0.
For example, 3760 is divisible by 10 because its units digit is 0.

(j) **Divisibility test by 11:** A number is divisible by 11 if the difference between the sum of its digits at even place and its digits at odd place is either 0 or 11.

For example, 45672 is divisible by 11 because the sum of digits 4,6 and 2 at odd place is $4 + 6 + 2 = 12$

The sum of digits 5,7 at even place is $5 + 7 = 12$

Hence, their difference ($12 - 12 = 0$) is zero.

So, 45672 is divisible by 11.

PRIME FACTORIZATION OR COMPLETE FACTORIZATION

Prime factorization of any number means presenting that number as a product of prime numbers.
For example, prime factorization of 40 can be done in the following way:

Prime factorization of 40

2	40
2	20
2	10
5	5
	1

Prime factorization of $40 = 2 \times 2 \times 2 \times 5$
 $= 2^3 \times 5$

(a) **Fundamental Theorem of Arithmetic:** Every composite number can be factored into prime numbers in only one way, except for the order of primes.

(b) **Common Factors:** Numbers which exactly divide two or more numbers are called their common factors.

All factors of 18 are: 1, 2, 3, 6, 9, 18

All factors of 24 are: 1, 2, 3, 4, 6, 8, 12, 24

Common factors of 18 and 24 are: 1, 2, 3, 6 as they exactly divide both 18 and 24.

HIGHEST COMMON FACTOR (H.C.F.)

Greatest common factor is the largest number which divides two or more numbers exactly.

Example (1): Let us find the H.C.F of two numbers 16 and 40.

Sol. All possible factors of 16 are: 1, 2, 4, 8, 16.

All possible factors of 40 are: 1, 2, 4, 5, 8, 10, 20, 40

Now the common factors of 16 and 40 are: 1, 2, 4, 8.

The highest of all these common factors is 8.

Similarly, there are two methods to find the greatest common factor:

(a) Prime Factorization Method: To find the HCF by prime factorization method, follow the following steps:

Step 1. Find the prime factorization of the given numbers.

Step 2. Find the common factors and circle them.

Step 3. Multiply the common factors to get the HCF.

Example: Let us now find the HCF of 72, 64 and 48.

Sol.

We can factorize the numbers as follows:

$$72 = 2 \times 2 \times 2 \times 3 \times 3$$

$$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

$$48 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$$

Take the common factor in all the given numbers.

$$72 = 2 \times 2 \times 2 \times 3 \times 3$$

$$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

$$48 = 2 \times 2 \times 2 \times 2 \times 2$$

Multiply the common factors to get the HCF.

$$\text{HCF of 72, 64 and 48} = 2 \times 2 \times 2 = 8$$

(b) Division method: In this method, the largest number among the given numbers is divided by the smallest number. If the remainder is zero, then the divisor is the greatest common factor. But if any number is left with a remainder, then the divisor is divided by the remainder and this process is repeated until the remainder is zero, the last divisor obtained is the greatest common factor of the given numbers. If the numbers are three, then the division process is done in the same way with the obtained greatest common factor and the third number.

Example: Find the HCF of 345 and 506.

Sol.

$$\begin{array}{r} 345 \overline{) 506} (1 \\ \underline{-345} \\ 161 \\ 161 \overline{) 345} (2 \\ \underline{-322} \\ 23 \\ 23 \overline{) 161} (7 \\ \underline{-161} \\ 0 \end{array}$$

Since, the last divisor is 23.

So, the HCF of 345 and 506 is 23.

LEAST COMMON MULTIPLE

LCM is the smallest number which is exactly divided by two or more numbers.

Similarly, there are two methods to find LCM:

(a) **Prime factorization method:** We can find LCM of any number by following the following steps-

Find the prime factorization of the given numbers.

LCM is the product of the factors of highest power.

or

Represent the factors with power, then multiply the numbers of highest power which is our desired LCM.

Example: Find the LCM of 84 and 96.

Sol. $84 = 2 \times 2 \times 3 \times 7$

$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$

LCM of 84, 96 = 672

(b) **Division method:** Find the LCM by following the following steps. is found out:

- Write the given numbers in one line separated by commas
- Divide these numbers by the smallest prime number that divides at least one of the given numbers.
- In another line write the quotients and numbers that are not divisible by prime numbers. Then repeat steps 2 and 3 and so on until all the numbers in one of the lines are 1.
- The LCM is found out by multiplying all the prime divisors and quotients other than 1.

Example: Find the LCM of 28, 36, 45 and 60.

Solution:

2	28, 36, 45, 60
2	14, 18, 16, 12
3	7, 9, 45, 15
3	7, 3, 15, 5
5	7, 1, 5, 5
	7, 1, 1, 1

L.C.M = $2 \times 2 \times 3 \times 3 \times 5 \times 7 = 1260$

RELATION BETWEEN LCM AND LCM

Let us take two numbers, say 16 and 24.

The LCM of 16 and 24 is 8.

The LCM of 16 and 24 is 48.

Since 8 is a factor of 48, we can say that the LCM of numbers is a factor of their LCM.

Product of LCM and LCM = $8 \times 48 = 384$

Product of numbers = $16 \times 24 = 384$

So, we can say that "The product of two numbers is equal to the product of their LCM and LCM."

Let a and b be two numbers then $a \times b = \text{HCF} \times \text{LCM}$

Example: HCF of two numbers is 29 and their LCM is 1160. If one number is 145, find the other number.

Sol. We know that,

Product of numbers = Product of their HCF and LCM

Required number $\times 145 = 29 \times 1160$

$$\text{Required number} = \frac{29 \times 1160}{145} = 232$$

