

FRACTIONS AND FUNDAMENTAL OPERATIONS



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“A fraction is a number that represents a part of a whole. This whole can be a single object or a group of objects.”

$\frac{5}{12}$ is a fraction. We read it as ‘Five-twelfth’.

TYPES OF FRACTIONS

a. Proper Fractions

A proper fraction is a fraction in which the numerator is smaller than the denominator.

Examples of proper fractions are $\frac{5}{12}, \frac{5}{8}, \frac{7}{33}, \dots$, etc.

b. Improper Fractions

An improper fraction is a fraction in which the numerator is greater than the denominator.

$\frac{7}{2}, \frac{8}{5}, \frac{17}{12}$ are all improper fractions.

c. Mixed Fractions

A fraction like $7\frac{2}{5}$ can also be expressed as $\frac{36}{5}$. So, numbers of the form $3\frac{1}{2}, 1\frac{3}{5}$, etc., are called mixed numbers or mixed fractions.

d. Unit fractions

Any fraction with 1 as the numerator is called a unit fraction, Thus, $\frac{1}{3}$ is called a unit fraction.

e. Equivalent Fractions

Consider the following fractions.

First one is $\frac{1}{2}$, second one is $\frac{2}{4}$, and the third one is $\frac{3}{6}$. Though written in different ways, they have the same value. Thus, fractions having equal value are called equivalent fractions.

There are two ways of obtaining equivalent fractions.

- i. By multiplying its numerator and denominator by the same number.

$$\frac{1}{2} = \frac{1 \times 2}{2 \times 2} = \frac{2}{4}, \frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6}, \frac{1}{2} = \frac{1 \times 4}{2 \times 4} = \frac{4}{8}$$
$$\Rightarrow \frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}$$

- ii. By dividing its numerator and denominator by the same number.

$$\frac{2}{4} = \frac{2 \div 2}{4 \div 2} = \frac{1}{2}, \frac{3}{6} = \frac{3 \div 3}{6 \div 3} = \frac{1}{2}, \frac{4}{8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$$

f. Like and Unlike Fractions

Two or more fractions with the same denominator are called like fractions, while fractions with different denominators are called unlike fractions.

For Ex. $\frac{7}{20}, \frac{13}{20}, \frac{9}{20}, \frac{11}{20}$ all are like fractions.

$\frac{3}{7}, \frac{8}{9}, \frac{3}{8}, \frac{6}{13}$ all are unlike fractions.

SIMPLIFICATION OF FRACTIONS AND THEIR SIMPLEST FORM

A fraction is said to be in its lowest term if the only common factor between the numerator and the denominator of the fraction is 1. That is, when the numerator and denominator are co-prime. This is also known as the simplest form or the standard form of the fraction.

For example, is in its lowest form.

A fraction can be expressed in its lowest term by following the steps given below.

- Find the Greatest Common Factor (HCF) of the numerator and denominator.
- Then divide both numbers by the HCF.
- The resulting fraction will be in its lowest term.

Example: Show that $\frac{7}{10}$ is in simplest form

Sol. Here, numerator = 7 and denominator = 10.

Factors of 7 are 1, 7.

Factors of 10 are 1, 2, 5, 10.

Common factors of 7 and 10 is 1 only.

HCF of 7 and 10 is 1.

Hence, $\frac{7}{10}$ is in the simplest form.

COMPARISON OF FRACTIONS

Comparison of like fractions: If the denominators of two given fractions are the same, then the fraction with the larger numerator is greater than the fraction with the smaller numerator.

For example: Compare two fractions $\frac{3}{7}$ and $\frac{5}{7}$.

Since $5 > 3$, therefore $\frac{3}{7} < \frac{5}{7}$.

Comparison of Unlike Fractions: Compare two fractions $\frac{7}{9}$ and $\frac{5}{12}$.

First, we convert $\frac{7}{9}$ and $\frac{5}{12}$ into their equivalent fractions.

LCM of 9 and 12 = 36

$$\frac{7}{9} = \frac{7 \times 4}{9 \times 4} = \frac{28}{36} \text{ and } \frac{5}{12} = \frac{5 \times 3}{12 \times 3} = \frac{15}{36}$$

Now, comparing, $\frac{28}{36}$ and $\frac{15}{36}$

$28 > 15$

therefore $\frac{7}{9} > \frac{5}{12}$;

OPERATIONS ON FRACTIONAL NUMBERS

Addition of Fractions

- To add two or more fractions having the same denominator, we add the numerators of the given fractions. The number thus obtained becomes the numerator of the required fraction and the denominator of this fraction is equal to the given fractions.

Example: Add the two fractions: $\frac{4}{18}$ and $\frac{7}{18}$.

$$\Rightarrow \frac{4}{18} + \frac{7}{18} = \frac{4+7}{18} = \frac{11}{18}$$

- First convert all the fractions into their equivalent fractions by using the common denominator which is the LCM of the denominators of the given fractions and then add them as mentioned before.
- To add two or more mixed numbers, we convert them into improper fractions and then add according to step 2.

Example: Add both the fractions - $\frac{2}{3}$ and $\frac{3}{14}$

$$\Rightarrow 1\frac{1}{8} + 3\frac{3}{8} = \frac{9}{8} + \frac{27}{8} = \frac{9+27}{8} = \frac{36}{8} = 4\frac{4}{8} = 4\frac{1}{2}$$

Subtraction of fractions

- To subtract two fractions with the same denominator, we subtract the smaller fraction from the larger fraction. The number thus obtained is the numerator of the required fraction and the denominator of this fraction is the denominator of the given fractions.

Example: Solve - $\frac{6}{11} - \frac{2}{11}$

Sol. $\frac{6}{11} - \frac{2}{11} = \frac{6-2}{11} = \frac{4}{11}$

- To subtract two fractions with opposite denominators, we convert both the fractions into equivalent fractions with common denominator and then subtract as explained earlier for fractions with like denominators.

Multiplication of Fractions:

- There are four simple rules to multiply fractions. First, multiply the numerators. Then, multiply the denominators of both the fractions to get the resultant fraction. Then, we need to simplify the fraction obtained to get the final answer.

Example: Multiply - $\frac{1}{3} \times \frac{3}{5}$

Sol.

Step - 1:

$$\frac{1 \times 3}{3 \times 5} = \frac{3}{15}$$

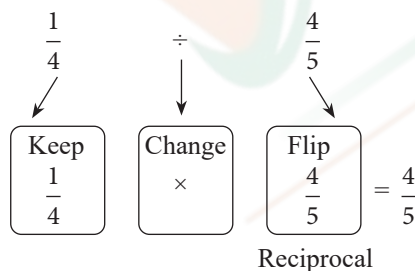
Step - 2:

$$\frac{3}{15} = \frac{1}{5}$$

$$\therefore \frac{1}{3} \times \frac{3}{5} = \frac{1}{5}$$

Division of Fractions:

- When we divide a fraction by another fraction, it is the same as multiplying the fraction by the reciprocal of the other fraction. The reciprocal of a fraction is a simple way of interchanging the numerator and denominator of a fraction. Observe the following image to learn the simple rule of dividing fractions-



Note: The reciprocal of any fraction is obtained by making the numerator of that fraction the denominator and the denominator the numerator.

DECIMALS AND FUNDAMENTAL OPERATIONS



DECIMAL NUMBERS

- Decimals are basically an extension of our number system.
- Decimals are basically simple forms of fractions whose denominators are 10, 100, 1000 etc.
- Decimals have two parts - integer part and decimal part.

(a) **Decimal places:** The number of digits contained in the decimal part of a decimal number is known as the number of decimal places.

Example: 3.75 has two decimal places and 85.325 has three decimal places.

(b) **Equal and Unequal Decimals:** Decimals having the same number of decimal places are called equal decimals, otherwise they are known as unequal decimals.

Example: 5.25, 15.04, 273.89 are equal decimals and 9.5, 18.235, 20.0254 etc. are unequal decimals.

(c) **Representation of decimal numbers on number line:** We have learnt how to represent whole numbers and fractions on the number line. Now we will understand how to represent decimal numbers on the number line.

Let us see how to represent 1.3 on a number line.

1.3 is more than 1 and less than 2.

$1.3 = 1 + 0.3$ i.e. 1 + one-tenth of 3.

So,



Draw a number line and mark the whole numbers 0, 1, 2, 3, ... on it.

Divide the part between 1 and 2 into 10 equal parts and take 3 parts for 3 tenths or 0.3 mark it as P.

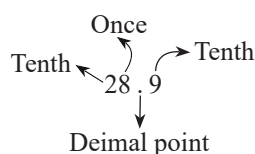
In the above figure P represents the number 1.3.

(d) **Division of a unit into ten equal parts:** If something is divided into 10 equal parts then each part is one tenth of the whole which is written as $\frac{1}{10}$

It is also written as 0.1 and read as 'one tenth' or 'decimal one or point one'.

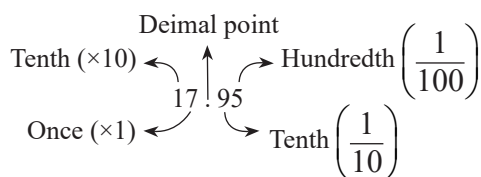
Thus 1 unit = 10 tenths.

For example, 0.5 is read as one tenth of 5.



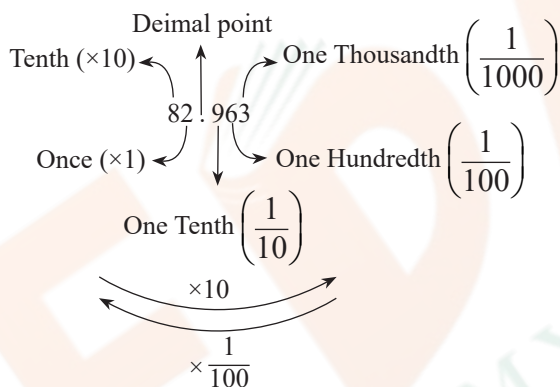
(e) **Division of a unit into a hundred equal parts:** If an object is divided into 100 equal parts, then each part is one hundredth of the whole. Which is written as $\frac{1}{100}$.

$\frac{1}{100}$ is also written as 0.01 and read as 'one hundredth' or 'decimal zero one' or zero decimal zero one'.



(f) **Division of a unit into a thousand equal parts:** If something is divided into 1000 equal parts then each part is one thousandth of the whole. And it is written as $\frac{1}{1000}$

$\frac{1}{1000}$ is also written as 0.001 and it is read as 'one thousandth' or 'decimal zero zero one' or 'zero decimal zero zero one'.



Example: Write the following in decimal form:

- (a) Eight tenths (b) Eight and three tenth

Sol.

- (a) 0.8 (b) 8.3

(g) **Expanded form of decimal numbers:** Let us study the following examples-

(a) $14.5 = 10 + 4 + \frac{5}{10} = 10 + 4 + 0.5$

(b) $49.08 = 40 + 9 + \frac{0}{10} + \frac{8}{100} = 40 + 9 + 0.08$

(h) **Change of Decimal Numbers**

i. Conversion of decimal numbers into common fractions: To convert a decimal number into a common fraction, express the decimal as a fraction with denominator 10 or power of 10 and then convert it to its lowest fraction which is the required fraction.

Example: (a) $0.75 = \frac{75}{100} = \frac{3}{4}$

(b) $0.125 = \frac{125}{1000} = \frac{1}{8}$

ii. Conversion of common fractions into decimal numbers: Divide the numerator by the denominator and write the quotient in decimal form, which is our desired decimal number.

Example: (a) $\frac{21}{8}$

(b) $2\frac{3}{8}$

(a) $\frac{21}{8} = 2.625$

(b) $2\frac{3}{8} = \frac{19}{8} = 2.375$

COMPARING DECIMAL NUMBERS

Comparing decimal numbers can be done using the following steps:

- First find the decimal numbers if they are given as fractions.
- Compare the integer parts of the numbers. The number with the larger integer part will also be larger. If the whole number parts are equal, go to the next step.
- Compare the last left digits of the decimal parts of the two numbers. The number with the largest left digit will be larger. If the last left digits of the decimal parts are equal, compare the next digits and repeat this process until you get different digits.

Example: Which is greater among 48.23 and 39.35?

Solution: The given decimals have different whole number parts, so we compare only the whole number parts

48.23 has whole number part 48.

39.35 has whole number part 39.

We know that $48 > 39$

So, $48.23 > 39.35$

OPERATIONS ON DECIMAL NUMBERS

(i) **Addition and Subtraction of Decimal Numbers:** Decimals can be added or subtracted using the following steps:

- Convert the given decimals into similar decimals.
- Write the decimals with their decimal points just below each other in a column so that tenths come under tenths, hundredths come under hundredths and so on for all the digits.
- Add or subtract whole numbers in the same way as we add or subtract whole numbers.
- Place the decimal point just below the other decimal points in the answer.

Example: Add 15.44, 7.524 and 25.

Solution: On converting the given decimals to equivalent decimals, we get 15.440, 7.524 and 25.000.

Now,

$$\begin{array}{r} 15.440 \\ + 7.524 \\ + 25.000 \\ \hline 47.964 \end{array}$$

MULTIPLICATION OF DECIMAL NUMBERS:

Multiplying decimals by 10, 100, 1000 etc.: To multiply decimals by 10, 100, 1000 etc., we use the following rules:

- When a decimal is multiplied by 10, the decimal point is shifted to the right by one place.
- When a decimal is multiplied by 100, the decimal point is shifted to the right by two places.
- When a decimal is multiplied by 1000, the decimal point is shifted to the right by three places and so on.

Example: Find the value of the following:

(a) 27.05×10 (b) 429.7×100

Sol. (a) $27.05 \times 10 = 270.5$ [Shifting the decimal point one place to the right]

(b) $429.7 \times 100 = 429.70 \times 100 = 42970$ [Shifting the decimal point two places to the right]

Multiplication of Decimal by Whole Number: Decimals can be multiplied by whole numbers using the following steps:

- Multiply the decimal without decimal point by the given whole number.
- Place the decimal point in the product where there are as many decimal places as there are in the given decimal.

(i) Division of decimal numbers

i. Dividing a decimal by 10, 100, 1000 etc.: A decimal can be divided by 10, 100, 1000 etc. using the following rules:

- When a decimal is divided by 10, the decimal point is shifted to the left by one place.
- When a decimal is divided by 100, the decimal point is shifted to the left by two places.
- When a decimal is divided by 1000, the decimal point is shifted to the left by three places.

Example: Divide: (a) 12.75 by 10

(a) $12.75 \div 10 = \frac{12.75}{10} = 1.275$ [Decimal shifted 1 place to the left]

ii. Dividing a decimal by a whole number: A decimal number can be divided by a whole number using the following steps:

- Check the whole number part of the dividend.
- If the integer part of the dividend or dividend is less than the divisor, put '0' in the unit place of the quotient, else go to previous step.
- Divide the integer part of the dividend.
- Place the decimal point to the right of the unit place in the quotient obtained initially.
- Divide the decimal part of the dividend by the divisor. If the dividend runs out of digits, put zeros to the right of the dividend and remainder each time and continue the process until the required quotient is obtained.

(j) **Division of Decimal by Decimal:** A decimal number can be divided by any decimal number using the following steps:

- Multiply the dividend and divisor by 10 or 100 or 1000 etc. to convert the divisor to a whole number.
- Divide the new dividend by the whole number obtained in step initially.

Example: Divide 0.00942 by 0.314.

Solution: We know that,

$$\frac{0.00942}{0.314} = \frac{0.00942 \times 1000}{0.314 \times 1000} = \frac{9.42}{314}$$

Hence, $0.00942 \div 0.314 = 0.03$

Example: The product of two decimal numbers is 1.5008. If one of the numbers is 0.56, find the other number.

Solution: The product of the given decimals = 1.5008.

One decimal = 0.56.

$$\text{Another Decimal} = 1.5008 \div 0.56 = \left(\frac{1.5008}{0.56} \times \frac{100}{100} \right) = \frac{150.08}{56} = 2.68$$

Hence, the second decimal number is 2.68.