

RATIO AND PROPORTION



RATIO

The comparison by division is called ratio. Ratio is usually denoted by the symbol ($:$).

If a and b ($b \neq 0$) are two quantities of the same kind, then the fraction $\frac{a}{b}$ is called the ratio of a to b , we write it as $a:b$.

Or $\frac{a}{b} = a \rightarrow$ antecedent $b \rightarrow$ consequent

In the ratio $a:b$, the first term is ' a ' and the second term is ' b '. A ratio is said to be in its simplest form if its two terms have no common factor other than 1.

Simplest form of ratio:

A ratio $\frac{a}{b}$ or $a:b$ is said to be in lowest or simplest form if a and b have no common factors except 1.

We can express any ratio in its lowest or simplest form by using the following steps.

- Write the ratio as a fraction.
- Divide the numerator and denominator by their HCF.
- The answer is a fraction in its lowest form; so, convert it to a ratio, which will be in its simplest form.
- For example:

$$(i) \quad 40:80 = \frac{40 \div 40}{80 \div 40} = \frac{1}{2} = 1:2$$

$$(ii) \quad 10000:8000 = \frac{10000 \div 2000}{8000 \div 2000} = \frac{5}{4} = 5:4$$

Example: Express the following ratios in their lowest form.

150: 400

Solution: $150:400 = \frac{150 \div 50}{400 \div 50} = \frac{3}{8} = 3:8$

Comparison of Ratios:

Ratios can be compared by following the following steps.

- Write the given ratios as fractions in simplest form.
- Find the LCM of the denominators of the fractions.

- Convert them into equivalent fractions having same denominators.
- Compare the numerators and arrange the fractions in ascending or descending order.
- Then the respective ratios are in the required order.

Example: Compare Both the Ratios: 5: 12 And 3: 8

Solution:

$$5:12 = \frac{5}{12}, 3:8 = \frac{3}{8}$$

$$\text{L.C.M. (8, 12)} = 24$$

$$5:12 = \frac{5}{12} \times \frac{2}{2} = \frac{10}{24}$$

$$3:8 = \frac{3}{8} \times \frac{3}{3} = \frac{9}{24}$$

$$\frac{9}{24} < \frac{10}{24}$$

$$\therefore \frac{3}{8} < \frac{5}{12} \text{ or } 3:8 < 5:12$$

Equivalent ratios:

Two or more ratios are considered equivalent if their simplest form is the same.

For example, $\frac{21}{35}, \frac{12}{20}$ are equivalent ratios because the lowest form of $\frac{21}{35}, \frac{12}{20}$ is $\frac{3}{5}$.

Dividing the whole in a certain ratio:

Follow the steps below:

- Find the sum of the terms of the ratio.
- First part = $\frac{\text{First Term}}{\text{Sum of the Ratio terms}} \times \text{Number to be distributed}$
- Second part = $\frac{\text{Second Term}}{\text{Sum of the Ratio terms}} \times \text{Number to be distributed}$

Example: Divide ₹4340 among A, B and C in such a way that.

$$\text{A's share: B's share: C's share} = \frac{1}{2} : \frac{1}{3} : \frac{1}{5}$$

Solution: According to the question, A's share: B's share: C's share = $\frac{1}{2} : \frac{1}{3} : \frac{1}{5}$

Taking LCM of 2, 3 and 5 = 30

$$\text{A's share: B's share: C's share} = \frac{1}{2} : \frac{1}{3} : \frac{1}{5} = \frac{1}{2} \times 30 : \frac{1}{3} \times 30 : \frac{1}{5} \times 30 = 15 : 10 : 6$$

$$\text{A's share} = \frac{15}{15+10+6} = \frac{15}{31} \times 4340 = ₹2100$$

$$\text{B's share} = \frac{10}{15+10+6} = \frac{10}{31} \times 4340 = ₹1400$$

$$\text{C's share} = \frac{6}{15+10+6} = \frac{6}{31} \times 4340 = ₹840$$

PROPORTION

If the ratio of the first to the second and the third to the fourth among four quantities is equal, then both these ratios are called proportion, and these four quantities are called proportional.

Example, $3: 5 = 9: 15$

The first and fourth terms are called outer or extreme terms. The second and third terms are called middle terms or mean.

In the subject of proportion, we can say that the product of outer terms = product of middle terms.

$\therefore$ Product of extreme terms = product of middle terms

$$\begin{array}{c} \downarrow \quad \downarrow \\ a : b = c : d \Rightarrow \frac{a}{b} = \frac{c}{d} \Rightarrow a \times d = b \times c \\ \uparrow \quad \uparrow \end{array}$$

Example: If $a: 30 :: 7: 15$, then find the value of a .

Solution: Product of extremes = $15a$

Product of mean = 30×7

Product of outside terms = Product of mean terms

$$\Rightarrow 15a = 30 \times 7$$

$$\therefore a = \frac{30 \times 7}{15} = 14$$

Continued Proportion:

Three numbers a, b, c are said to be in continued proportion if a, b, b, c are in proportion.

Thus, if a, b, c are in continued proportion, then

a, b, b, c are in proportion, i.e., $a : b :: b : c$

Product of extreme terms = Product of mean terms

$$\Rightarrow a \times c = b \times b$$

$$\Rightarrow ac = b^2$$

$$\Rightarrow b^2 = ac.$$

Mean proportion:

If a, b, c are in continuous or in constant proportion, then b is called the mean proportion between a and c .

Clearly, if b is the mean proportional between a and c , then $b^2 = ac$.

Third proportion:

If $a : b = b : c$, then c is called the third proportional to a and b .

Note:

Mean proportional of $a, b = \sqrt{ab}$

Third proportional of $a, b = \frac{b^2}{a}$

Fourth proportional of $a, b, c = \frac{bc}{a}$

UNITARY METHOD

In Unitary method, it involves finding the value of a single unit (one unit) and then using that value to determine the value of the required number of units.

Steps to be followed in Unitary Method.

- Know what to find.
- Find the value of the unit quantity as follows:
 - Value of one unit = $\frac{\text{Given value}}{\text{Total number of items}}$
 - Multiply this result by the number of units required.
 - Value of many = Value of one $\times$ Number of units required.

Example: Raj earns ₹1500 in 10 days. How much will he earn in 45 days?

Solution: Earning in 10 days = ₹1500

Earning in 1 day = $\frac{1500}{10} = ₹150$

$\therefore$ Earning in 45 days = ₹150 $\times$ 45 = ₹6750

Thus, Raj earns = ₹6750 in 45 days