

AVERAGE



AVERAGES

The average or arithmetic mean of several quantities of the same kind is the sum of the quantities divided by the number of the quantities.

- Arithmetic average is used for all averages such as: average income, average profit, average age, average marks etc.
- Average is the sum of all quantities of a series of items divided by the total number of items.
- In a series of items of the same kind,

$$\text{Average} = \frac{\text{The sum of all the given observations}}{\text{Number of Observations}}$$

OR

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$$

- To calculate the sum of observations, they must be in the same unit.

Some important results:

- Average of first n even natural numbers $= (n + 1)$
- Average of first n odd natural numbers $= (n)$
- Average of first n natural numbers $= \frac{(n+1)}{2}$

Average When a New Value is Added

When a new number is added to a group, the new average can be found without calculating everything again.

Formula:

$$\text{New Average} = \frac{(\text{Old Average} \times \text{Old Number of Terms}) + \text{New Value}}{\text{New Number of Terms}}$$

Example: Average of 4 numbers is 20. A 5th number, 30, is added. Find the new average.

Sol: Total of 4 numbers $= 4 \times 20 = 80$

Add new number: $80 + 30 = 110$

New number of terms $= 5$

New average $= 110 \div 5 = 22$

Average When a Value is Removed

If one number is removed, the average of the remaining group can be found similarly.

$$\text{New Average} = \frac{(\text{Old Average} \times \text{Old Number of Terms}) - \text{Removed Value}}{\text{New Number of Terms}}$$

Example: Average of 5 numbers is 18 . One number, 28, is removed. Find the new average.

Sol: Total = $5 \times 18 = 90$

After removing 28 $\rightarrow 90 - 28 = 62$

New number of values = 4

New average = $62 \div 4 = 15.5$

Finding the Missing Number When Average is Known

Example: The average of 4 numbers is 25 . Three of the numbers are 24, 28 , and 22 . Find the 4 th number.

Sol: Total = $4 \times 25 = 100$

Known sum = $24 + 28 + 22 = 74$

Missing number = $100 - 74 = 26$

AVERAGE OF CONSECUTIVE NUMBERS

For n consecutive numbers (like 1,2,3... or 11,12,13...), the average is the middle number.

Example: Find the average of 5,6,7,8,9

Sol: Middle number = 7

So, Average = 7

Average of Evenly Spaced Numbers

If numbers have equal difference (e.g., 2, 4, 6, 8), the average is the average of first and last number.

$$\text{Average} = \frac{\text{First} + \text{Last}}{2}$$

Example:

Average of 10, 20, 30, 40

Sol: $(10 + 40) \div 2 = 25$

Using Change in Average to Find Value

If the average increases or decreases when a new person/value is added, we can find the new value.

Example: The average of 5 numbers is 60 . A sixth number is added, and the new average becomes 62. Find the sixth number.

Sol: Total before = $5 \times 60 = 300$

New total = $6 \times 62 = 372$

Sixth number = $372 - 300 = 72$